Increasing operational reliability in diverse catchment conditions through the application of a flexible modelling approach

F. Fenicia, D. Kavetski, H. H. G. Savenije, G. Schoups, J. Juilleret, L. Pfister, M. P. Clark, J. Freer

Outline

- Intro hydrological modelling
- Flexible models
- Case study: 3 headwater catchments in Luxembourg
  - Experimental insights
  - Modelling work
- Conclusions

Introduction: Hydrological modeling

- Describe water cycle dynamics at catchment scale
  - Lumped vs distributed, conceptual vs physically-based, etc.
- Scientific and engineering hydrology
  - Process understanding, water resource assessment, climate studies
- Increasingly used in environmental management and decision-making across many levels
- Robustness and ease of application very important
- Here, we focus on lumped conceptual models: do not attempt to directly resolve “small-scale” physics
  - Computationally fast
  - Can often capture dominant dynamics without requiring extensive data such as for distribute
- Study also relevant to other types of hydrological and broader environmental models (more later)

How should hydrological models be developed?

From “Hydrology: The Primer” by K. Beven

1. Develop a qualitative “perceptual” model
   – Decide which processes are dominant, scale of model, etc
2. Develop a quantitative “conceptual“ model
   – Mathematical description of the conceptual model
   – Additional simplifications (eg, soil homogeneity, etc)
   – Provides “governing equations“ (physical or conceptual)
   – “Model structural errors“ due to assumptions/simplifications
3. Numerical solution / computer algorithm
   – Usually governing equations not analytically tractable
   – “Numerical errors” due to numerical approximations
4. Estimate parameters (a prior or calibration)
5. Evaluate model against data / scrutinize hypotheses
6. Refine model if necessary -> model-building cycle

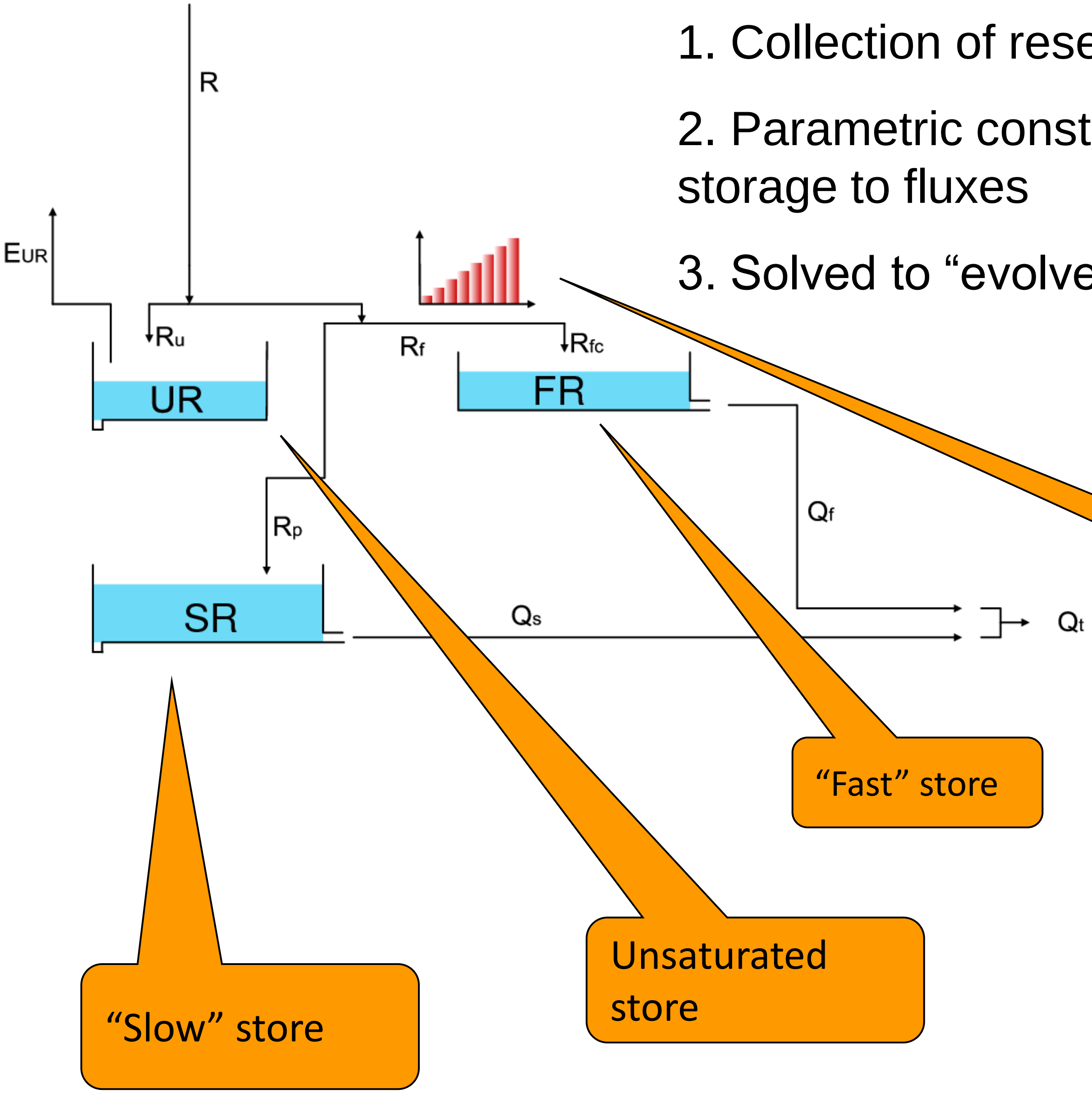
Some “pragmatic” shortcuts in model development …

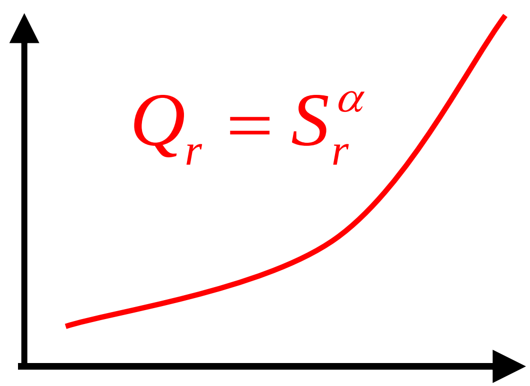
often at the expense of model mathematics …

1. Skip the formulation of governing state equations and go directly to the computer algorithm
   – Quite common, especially in “conceptual” models which “just” move water across a few buckets
   – But can also happen in more complex models, eg, in the Sacramento model, the fluxes are processed sequentially, eg, runoff first, then baseflow, then evaporation, etc
2. Use simplistic numerical techniques
   – “Explicit“ time stepping very common in conceptual hydrological models: $S_{n+1} = S_n - Q(S_n)$ ... *Models are simplistic anyway, right?*
3. Neglect to refine model structure
   – ‘One model fits all‘ vs. ‘Flexible models‘
   – Poor guidance on model development

Conceptualization of rainfall-runoff models

1. Collection of reservoirs in series/parallel
2. Parametric constitutive functions relating storage to fluxes
3. Solved to “evolve” storages thru time





A careful mathematical perspective

- Continuous-time form (eg, Kavetski et al, WRR2003)
- Sets of (coupled) differential equations

$$\mathrm{d}\boldsymbol{S}(t)/\mathrm{d}t = \boldsymbol{g}_S\left(\boldsymbol{S}(t), \boldsymbol{P}(t) \,|\, \boldsymbol{\theta}\right) \quad \ldots\ldots\ \text{(a)}$$

$$Q(t) = \boldsymbol{g}_Q\left(\boldsymbol{S}(t), \boldsymbol{P}(t) \,|\, \boldsymbol{\theta}\right) \quad \ldots\ldots\ \text{(b)}$$

S = states, θ = parameters, *P* = forcings, *Q* = responses

- For example, VIC model (Wood, 1992)

$$\mathrm{d}S(t)/\mathrm{d}t = P\left[1 - S / S_{\max}\right]^{\alpha} - kS^{\beta} - E(S)$$

… though not always cast in this form …

Numerical solution/implementation aspects

- Analytical solution of water balances is usually impossible when the model has nonlinear fluxes wrt states
- Numerical approximations are employed
  - Explicit Euler scheme (widely used in conceptual hydrology)

$$\boldsymbol{S}_{EE}^{n+1} = \boldsymbol{S}^{n} + \Delta t \; \boldsymbol{g}(\boldsymbol{S}^{n})$$

  - Implicit Euler scheme (common in engineering/groundwater)

$$\boldsymbol{S}_{IE}^{n+1} = \boldsymbol{S}^{n} + \Delta t \; \boldsymbol{g}(\boldsymbol{S}_{IE}^{n+1})$$

  - Fixed-step methods introduce numerical approximation errors
  - Adaptive numerical solutions (common in applied maths)

Though seemingly mundane, the numerical approximation technique has a profound impact on model behavior..

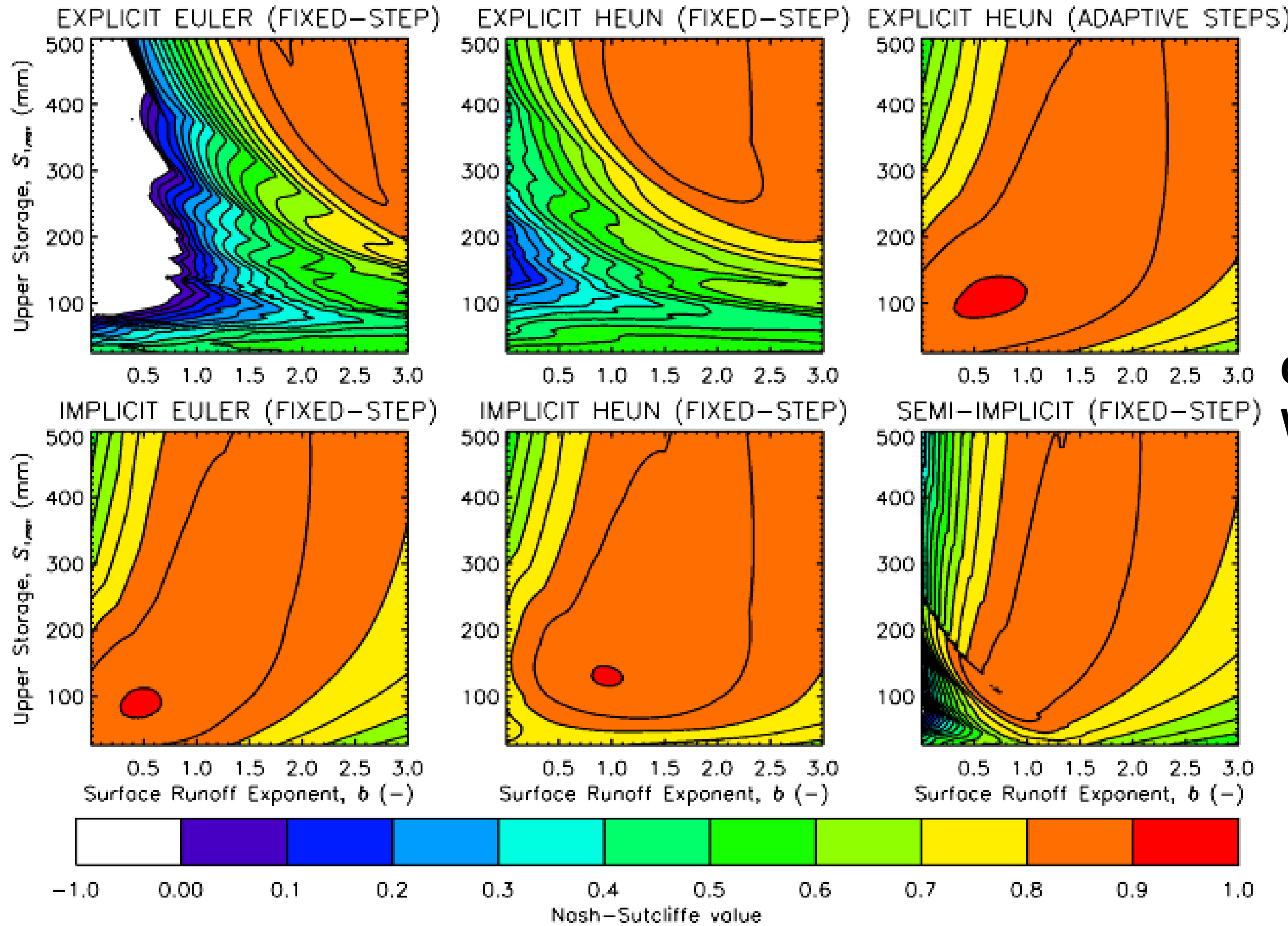
… yes, even when data is inexact and model is poor!

Objective function complexity: Numerical artefacts?

Same model structure, same data, same objective function

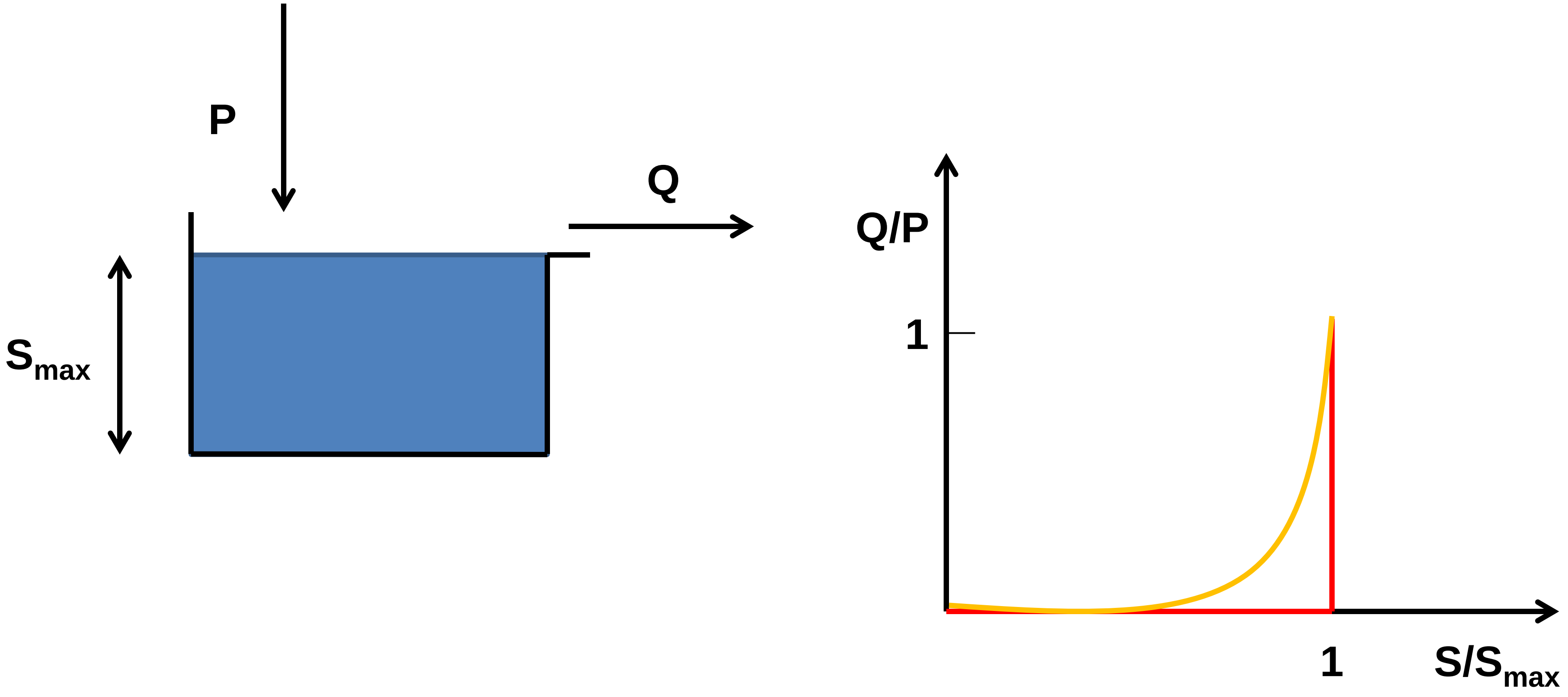
BUT different time stepping schemes

Explicit: $S_{t+1} = S_t - dt * Fluxes(S_t)$ or Implicit: $S_{t+1} = S_t - dt * Fluxes(S_{t+1})$



Clark and Kavetski WRR 2010

“Smoothing” of constitutive relationships



WATER RESOURCES RESEARCH, VOL. 46, W10510, doi:10.1029/2009WR008894, 2010

Ancient numerical daemons of conceptual hydrological modeling: 1. Fidelity and efficiency of time stepping schemes

Martyn P. Clark[1] and Dmitri Kavetski[2]

Received 12 November 2009; revised 22 March 2010; accepted 16 April 2010; published 8 October 2010.

[1] A major neglected weakness of many current hydrological models is the numerical method used to solve the governing model equations. This paper thoroughly evaluates several classes of time stepping schemes in terms of numerical reliability and computational efficiency in the context of conceptual hydrological modeling. Numerical experiments are carried out using 8 distinct time stepping algorithms and 6 different conceptual rainfall-runoff models, applied in a densely gauged experimental catchment, as well as in 12 basins with diverse physical and hydroclimatic characteristics. Results show that, over vast regions of the parameter space, the numerical errors of fixed-step explicit schemes commonly used in hydrology routinely dwarf the structural errors of the model conceptualization. This substantially degrades model predictions, but also, disturbingly, generates fortuitously adequate performance for parameter sets where numerical errors compensate for model structural errors. Simply running fixed-step explicit schemes with shorter time steps provides a poor balance between accuracy and efficiency: in some cases daily-step adaptive explicit schemes with moderate error tolerances achieved comparable or higher accuracy than 15 min fixed-step explicit approximations but were nearly 10 times more efficient. From the range of simple time stepping schemes investigated in this work, the fixed-step implicit Euler method and the adaptive explicit Heun method emerge as good practical choices for the majority of simulation scenarios. In combination with the companion paper, where impacts on model analysis, interpretation, and prediction are assessed, this two-part study vividly highlights the impact of numerical errors on critical performance aspects of conceptual hydrological models and provides practical guidelines for robust numerical implementation.

Intro flexible models

Modelling choices

- Physically based hydrological models require large amounts of data (e.g. soil characteristics, bedrock topography, hydraulic properties, etc.)
- When this data is not available, the application of physically based models is questionable
- Conceptual models are therefore preferred

Conceptual models

- Operational purposes (easy to operate, easy to calibrate, computationally fast)
- Research purposes (hypothesis testing, interpretable building blocks)

Fix or flex?

- Until recently, hydrological modelling has focused on the development of a fixed model structure
- Fixed models condense experience across different places, facilitate comparisons, etc...
- But experience has shown that (i) models often need adaptations, and (ii) conceptual models continue to proliferate
- Flexible models answer to the need of (i) improving hypotheis testing, and (ii) adapting to diverse conditions (data availability, catchments, case studies)

The SuperFlex framework

WATER RESOURCES RESEARCH, VOL. 47, W11510, doi:10.1029/2010WR010174, 2011

Elements of a flexible approach for conceptual hydrological modeling: 1. Motivation and theoretical development

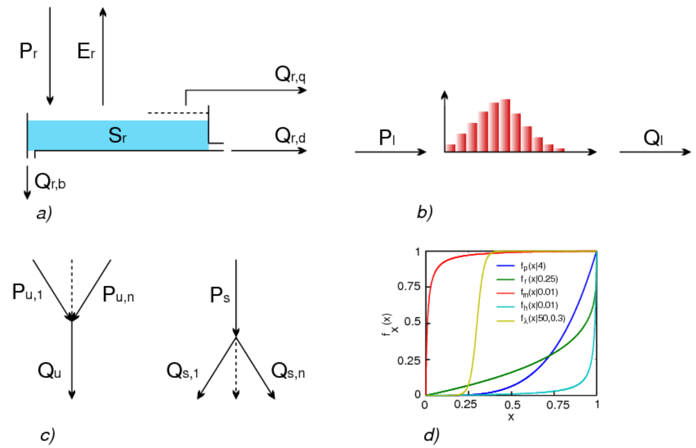
Fabrizio Fenicia,[1,2] Dmitri Kavetski,[3] and Hubert H. G. Savenije[2]

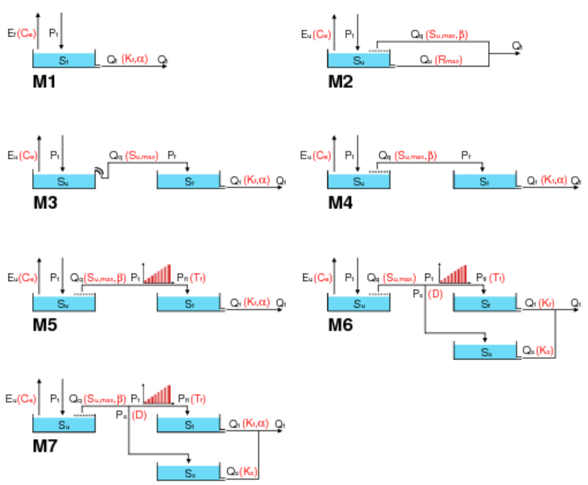
Received 27 October 2010; revised 19 September 2011; accepted 19 September 2011; published 11 November 2011.

[1] This paper introduces a flexible framework for conceptual hydrological modeling, with two related objectives: (1) generalize and systematize the currently fragmented field of conceptual models and (2) provide a robust platform for understanding and modeling hydrological systems. In contrast to currently dominant "fixed" model applications, the flexible framework proposed here allows the hydrologist to hypothesize, build, and test different model structures using combinations of generic components. This is particularly useful for conceptual modeling at the catchment scale, where limitations in process understanding and data availability remain major research and operational challenges. The formulation of the model architecture and individual components to represent distinct aspects of catchment-scale function, such as storage, release, and transmission of water, is discussed. Several numerical strategies for implementing the model equations within a

The SuperFlex framework

- Generic elements
  - Reservoir
  - Lag function
  - Connections
- Systematizes the field of conceptual modelling
- Aids hypothesis testing





Research question

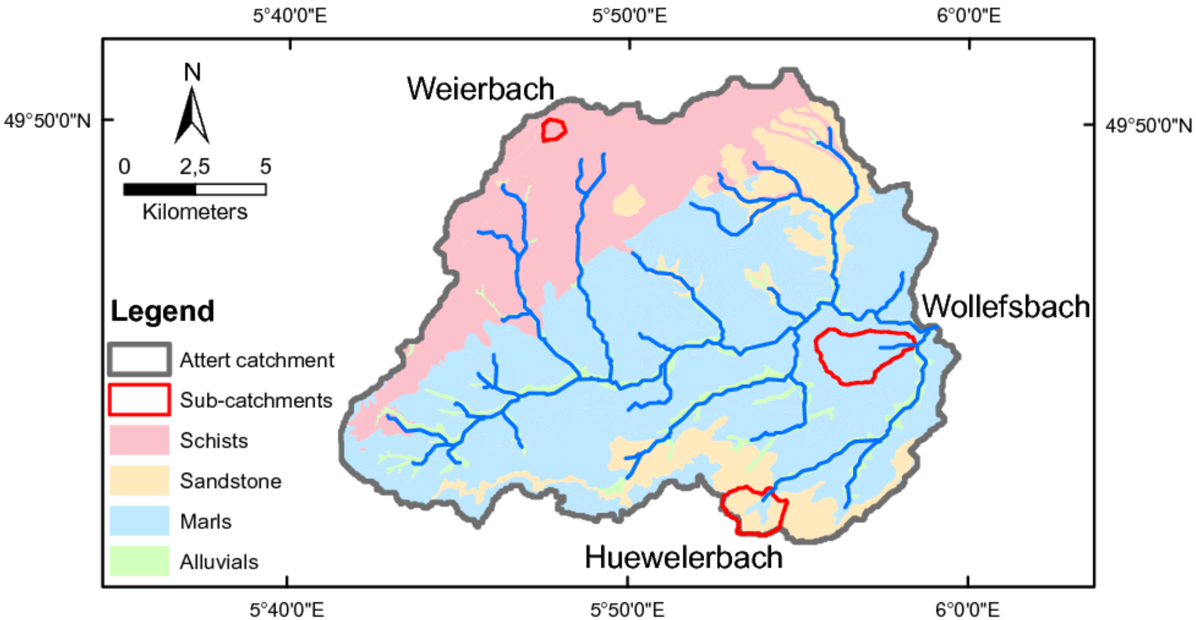
Correspondence between catchment structure and conceptual model structure

- Poorly understood
  - ‘One model fits all‘ vs. ‘Flexible models‘
  - Poor guidance on model development
- Connects to major research themes
  - Catchment classification
  - PUB
  - Development of new theories of hydrology at the catchment scale

Case study in Luxembourg

3 headwaters in Luxembourg

- Huewelerbach
  - 2.7 km^2,
  - Forest, grassland
  - Sandstone
- Weierbach
  - 0.42 km^2
  - Forest
  - Schist
- Wollefsbach
  - 4.5 km^2
  - Grassland, cropland
  - Marls



Land use and geology

1. Huewelerbach

Huewelerbach - sandstone

Huewelerbach - sandstone

Huewelerbach - sandstone

2. Weierbach

Weierbach- schist

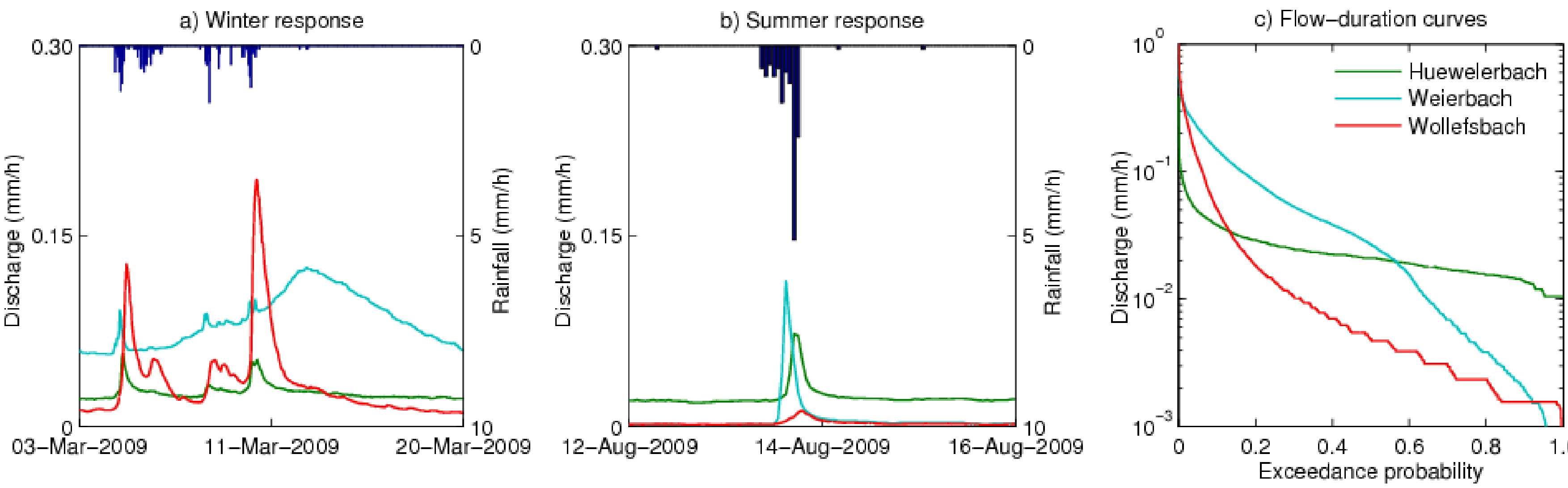
Weierbach- schist

2. Wollefsbach

Wollefsbach - Marls

Hydrological response

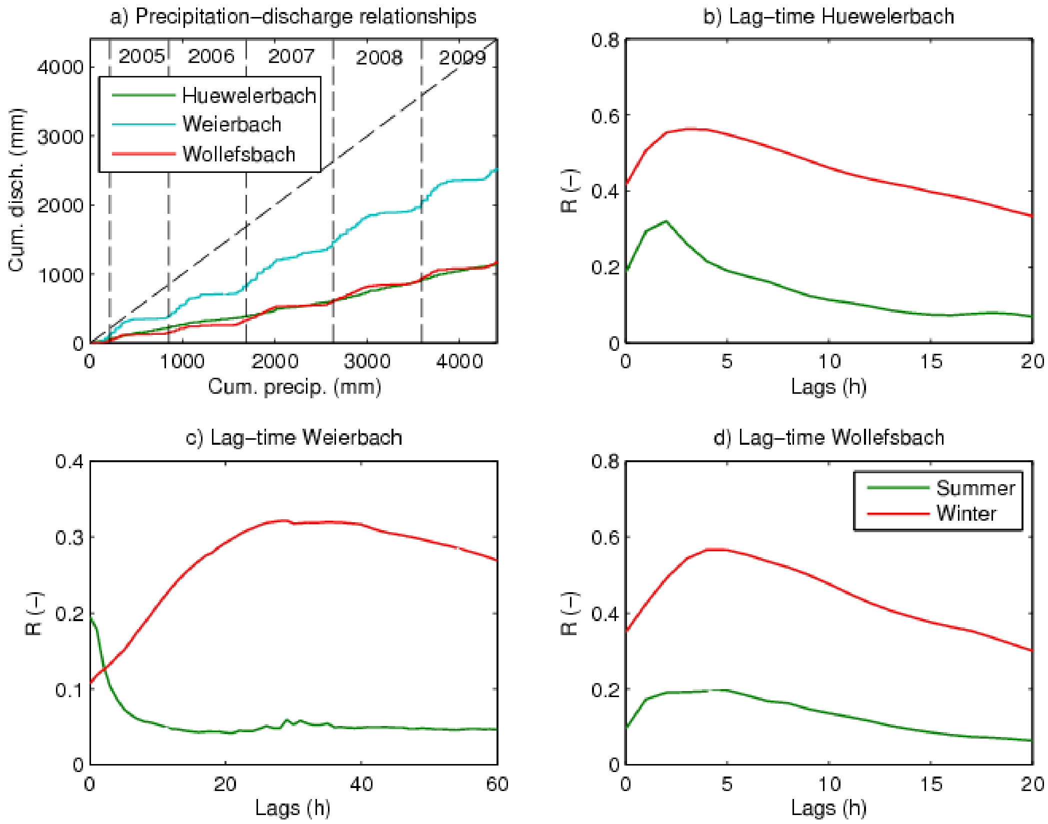
Hydrograph



- Huewelerbach: stable, constant baseflow
- Weierbach: lag in winter, threshold-like
- Wollefsbach fast response, threshold-like

Rainfall-discharge summaries

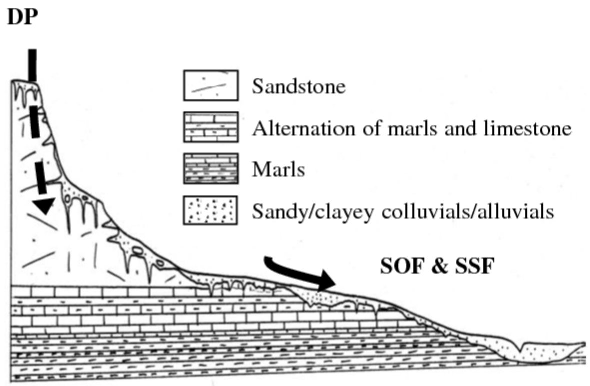
- Linear P-Q relation for Huewelerbach
- Threshold P-Q relation for Weierbach and Wollefsbach
- Differences in wet and dry lag times for Weierbach



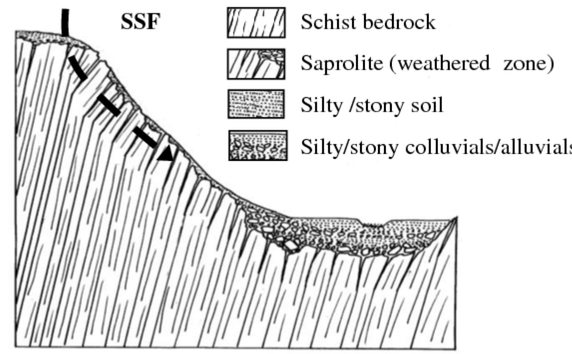
ERT measurements, tranches, soil samples, etc.

Perceptual models

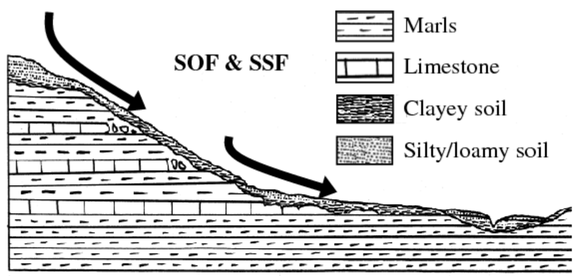
Perceptual models



a) Huewelerbach catchment (sandstone lithology)



b) Weierbach catchment (schistose lithology)

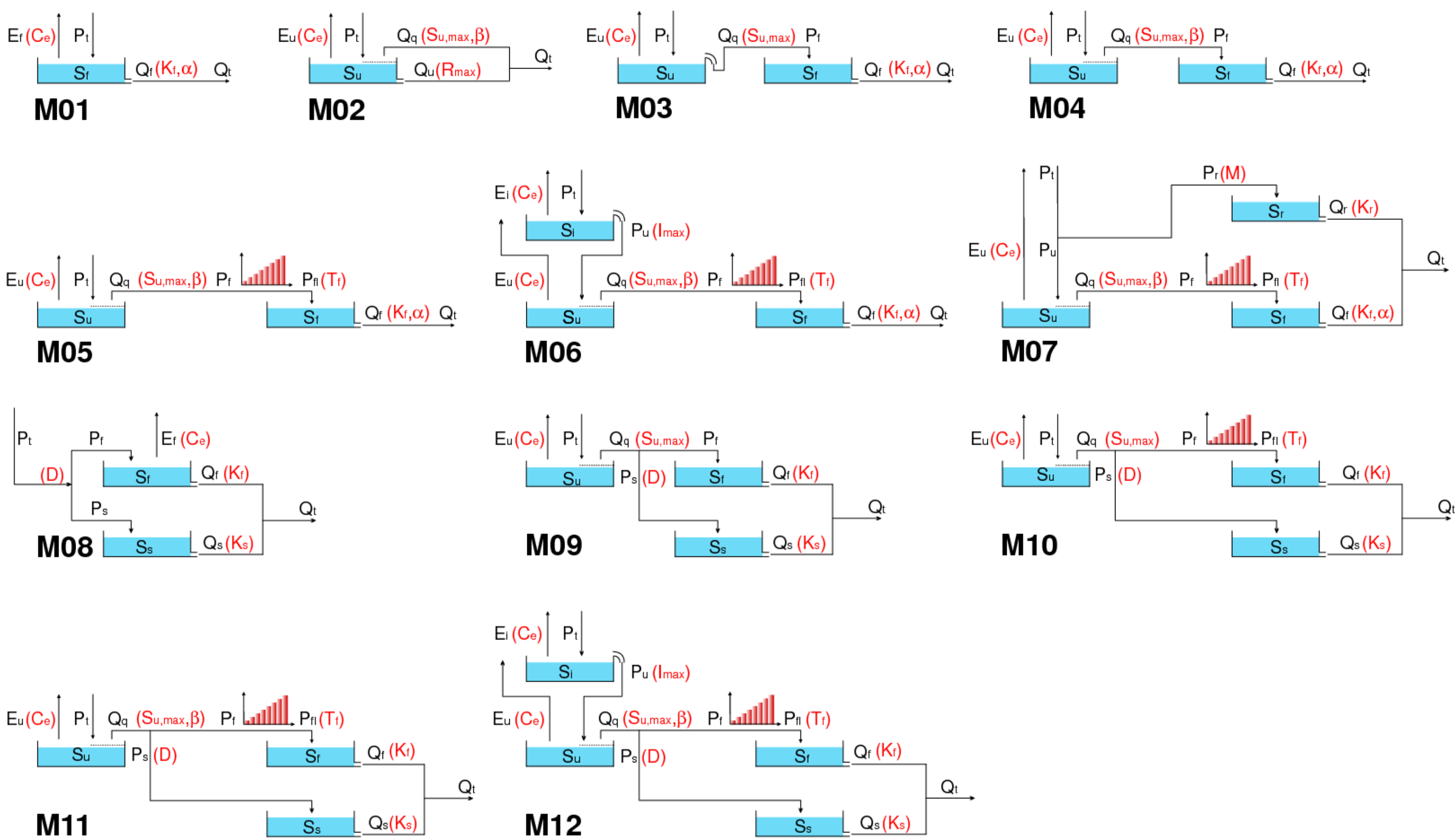


c) Wollefsbach catchment (marly lithology)

SOF: Saturated Overland Flow
SSF: Subsurface Flow
DP: Deep Percolation

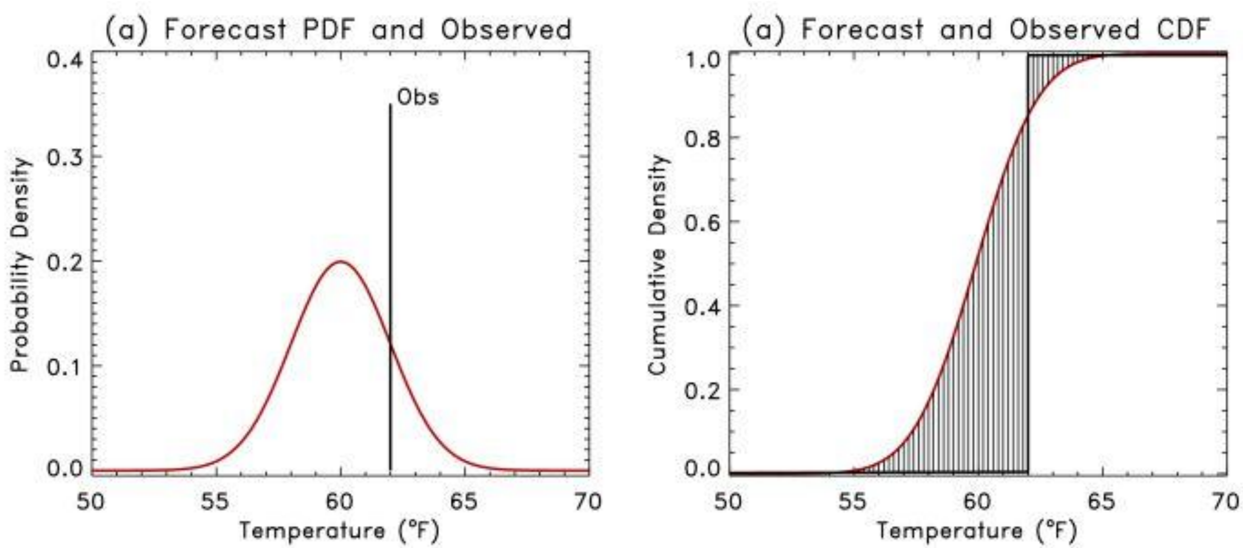
Modelling

12 model structures (SuperFlex)

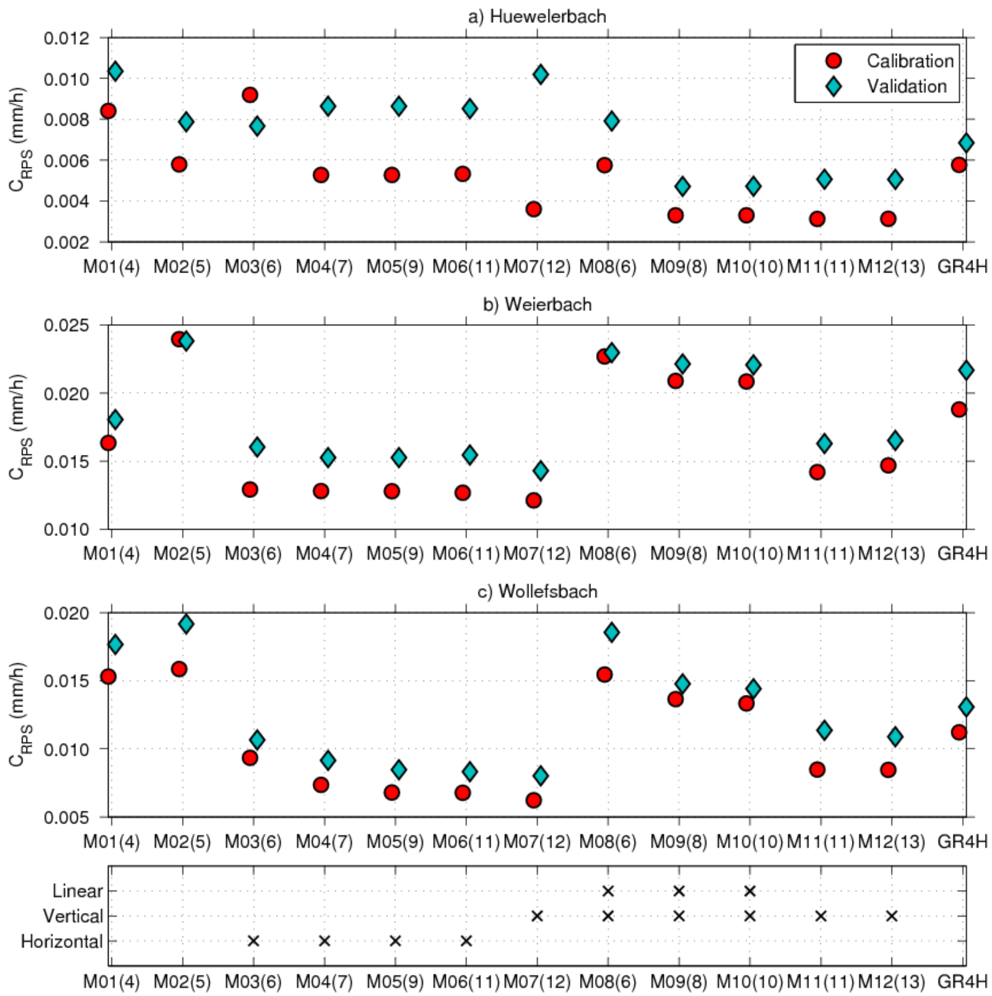


Model Calibration - Evaluation

- Weighted least square calibration
  - Error proportional to discharge (heteroscedastic)
- Evaluation using the Continuous Rank Probability Score (CRPS)



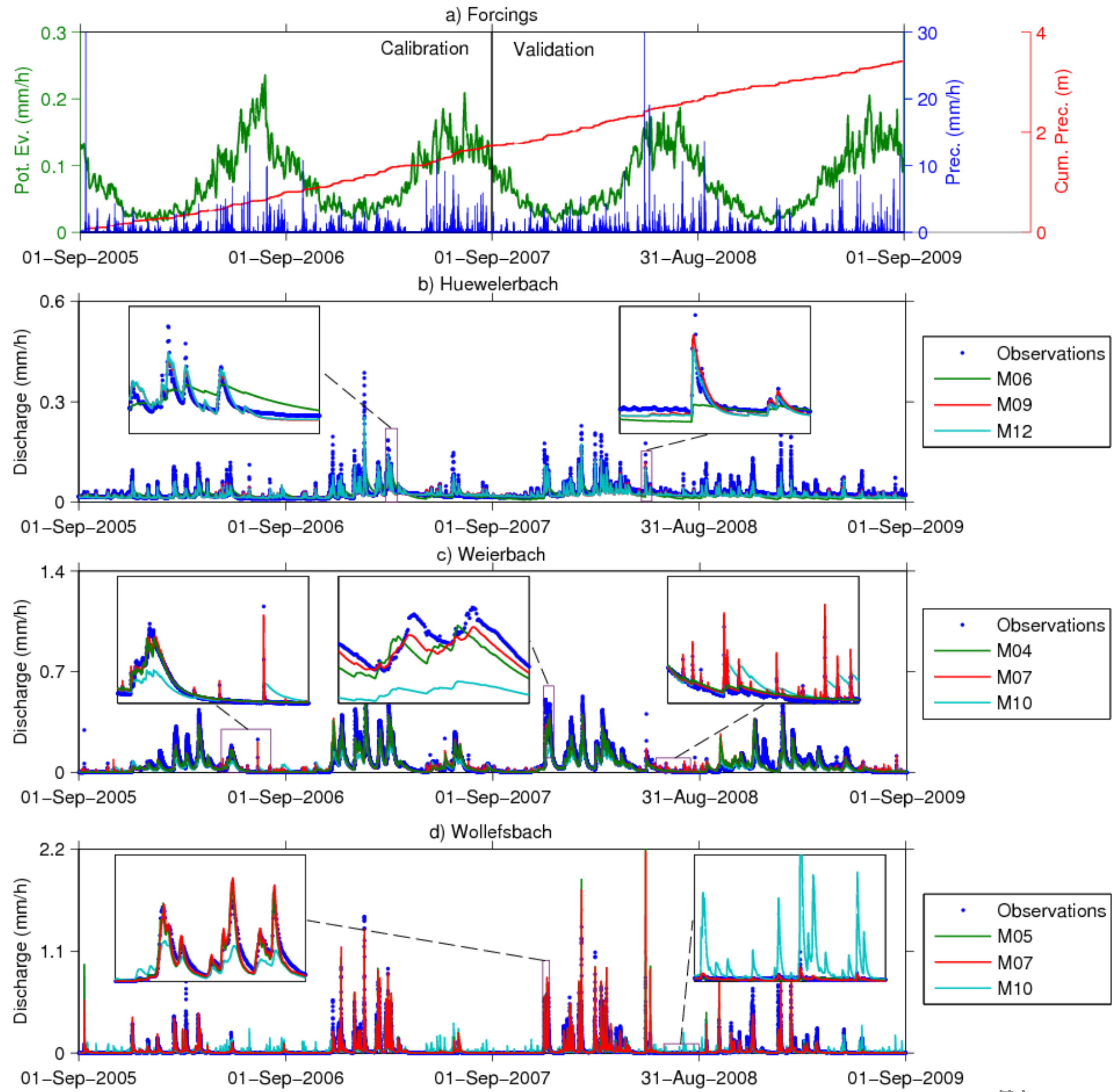
Results



- Huewelerbach is well simulated by vertical structures and linear models
- Weierbach and Wollefsbach are well simulated with horizontal structures and threshold models

Hydrograph simulations

- Effect of lag function on Weierbach (M07)
- Linear models on all catchments (M09 and M10 are linear)



Conclusions

- Experimenting and modelling contribute differently to the overall picture of “How a catchment works“
- For these 3 headwater catchments, we could find a meaningful correspondence between catchment structure and conceptual model structure
- Similar model concepts may have very different experimental interpretation
- What happens in other catchment, with mixed geologies, larger areas, etc... ?

21-25 May 2012
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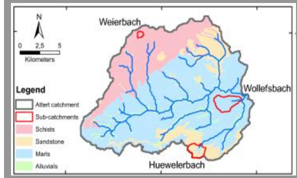

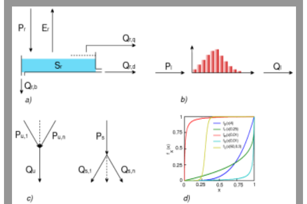

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Hydro2012 short course

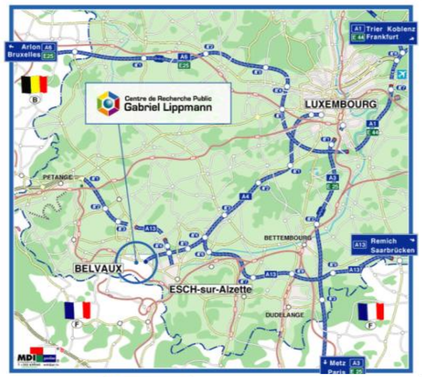
HYDRO2012 short course

Personnel

- Organizing committee
  - Fabrizio Fenicia (CRP – Gabriel Lippmann)
  - Laurent Pfister (CRP – Gabriel Lippmann)
  - Dmitri Kavetski (University of Adelaide, Australia)
- Lecturers
  - Fabrizio Fenicia (CRP – Gabriel Lippmann)
  - Dmitri Kavetski (University of Adelaide, Australia)
- Invited lecturers
  - Martyn Clark (NCAR, Boulder)
  - Benjamin Renard (IRSTEA, Lyon)
  - Hubert Savenije (Delft University of Technology)

Venue and contact point

The workshop will be held at the CRP-Gabriel Lippmann:
41, rue du Brill L-4422 Belvaux I Luxembourg
Email: hydrocourse@lippmann.lu
Website: http://hydrocourse.lippmann.lu



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